Exclusion of the Longitudinal Polarisation of a Massive Gauge Bosons

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Abstract

It is shown that a massive gauge boson in the initial or final states in scattering processes have two transverse polarisations and no longitudinal polarisation.

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For a long time, massive gauge bosons appear to be very awkward. One of the main accusations is that their explicit mass term spoils the renormalizability of the Yang-Mills gauge theories. For studies on this subject see refs. [1-8] where [1] includes our recent work. In this paper we investigate a related but more extensive problem: a massive gauge boson was thought to have longitudinal polarizition and thus often leads to 'catastrophes'. We will prove that a massive gauge boson in the initial or final states has no longitudinal polarization at all. Thereby all the ideas based on the existence of such a kind of longitudinal polarization should be abandoned.

No matter what kind of massive gauge theory is, the gauge bosons in initial or final states are always free particles described by a massive Abelian gauge theory. It is therefore enough to start from the following Lagrangian

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}M^2 A_\mu A^\mu \,, \tag{1}$$

where A_μ stands for the gauge fields and

$$F_{\mu\nu}(x) = \partial_\mu A_\nu(x) - \partial_\nu A_\mu(x)\,. \tag{2}$$

As it is well known, the mass term leads to the Lorentz condition

$$\partial^\mu A_\mu(x) = 0\,. \tag{3}$$

The quantization under such a constraint condition can be carried out by following the path-integral method of ref. [1]. Furthermore, according to refs. [1,5] one can regard the theory as a gauge invariant theory by replacing the mass term with a gauge invariant quantity $\widetilde{\mathcal{L}}_{AM}$ which is equal to the original expresion when the Landau gauge is chosen. Thus, the effective Lagrangian in the $\xi-$gauge has the form

$$\mathcal{L}_{eff} = -\frac{1}{2\xi}\big(\partial^\mu A_\mu(x)\big)^2 + \mathcal{L}^{(C)} - \frac{1}{2}\big(\partial_\mu A_\nu\big)\big(\partial^\mu A^\nu\big) + \widetilde{\mathcal{L}}_{AM}\,, \tag{4}$$

where ξ is a real parameter and $\mathcal{L}^{(C)}$ is the ghost Lagrangian

$$\mathcal{L}^{(C)}(x) = -\big(\partial_\mu \overline{C}(x)\big)\partial^\mu C(x)\,. \tag{5}$$

Here, we will employ the Gupta-Bleuler method. Namely, we first use Eq. (3) to modify the Lagrangian as

$$\mathcal{L}_{GB} = -\frac{1}{2}\big(\partial_\mu A_\nu\big)\big(\partial^\mu A^\nu\big) + \frac{1}{2}M^2 A_\mu A^\mu + \mathcal{L}^{(C)}\,, \tag{6}$$

and perform a canonical quantization and define the field operators and the Hamiltonian opereator H_0 by paying no attention to the constraint condition. We then define the physical state subspase with the help of the physical eigen states of H_0. Assume that the operater of SCS is anti-hermitian and therefore that of $\overline{C}$ is anti-hermitian. Let $\omega_1(x)$, $\omega_2(x)$ stand for $\overline{C}$, $iC(x)$ and Π_1, Π_2, Π_μ stand for the canonical momenta conjugate to $\omega_1(x)$, $\omega_2(x)$, A^μ respectively. Thus the operators of these qiantities in the Schrodinger picture are

$$A_\mu(\mathbf{x}) = \int \frac{d^3k}{\sqrt{2\omega(2\pi)^3}}\Big[a_\mu(\mathbf{k})e^{i\mathbf{k}\cdot\mathbf{x}} + a_\mu^\dagger(\mathbf{k})e^{-i\mathbf{k}\cdot\mathbf{x}}\Big]\,, \tag{7}$$

$$\Pi_\mu(\mathbf{x}) = \int \frac{id^3k}{\sqrt{(2\pi)^3}}\sqrt{\frac{\omega}{2}}\Big[a_\mu(\mathbf{k})e^{i\mathbf{k}\cdot\mathbf{x}} - a_\mu^\dagger(\mathbf{k})e^{-i\mathbf{k}\cdot\mathbf{x}}\Big]\,, \tag{8}$$

$$\omega_1(\mathbf{x}) = \int \frac{d^3k}{\sqrt{2|\mathbf{k}|(2\pi)^3}}\Big[\overline{C}(\mathbf{k})e^{i\mathbf{k}\cdot\mathbf{x}} + \overline{C}^\dagger(\mathbf{k})e^{-i\mathbf{k}\cdot\mathbf{x}}\Big]\,, \tag{9}$$

$$\Pi_1(\mathbf{x}) = \int \frac{(-i)d^3k}{\sqrt{(2\pi)^3}}\sqrt{\frac{|\mathbf{k}|}{2}}\Big[C(\mathbf{k})e^{i\mathbf{k}\cdot\mathbf{x}} + C^\dagger(\mathbf{k})e^{-i\mathbf{k}\cdot\mathbf{x}}\Big]\,, \tag{10}$$

$$\omega_2(\mathbf{x}) = \int \frac{(-i)d^3k}{\sqrt{2|\mathbf{k}|(2\pi)^3}}\Big[C(\mathbf{k})e^{i\mathbf{k}\cdot\mathbf{x}} - C\dagger(\mathbf{k})e^{-i\mathbf{k}\cdot\mathbf{x}}\Big]\,, \tag{11}$$

$$\Pi_2(\mathbf{x}) = \int \frac{d^3k}{\sqrt{(2\pi)^3}}\sqrt{\frac{|\mathbf{k}|}{2}}\Big[\overline{C}(\mathbf{k})e^{i\mathbf{k}\cdot\mathbf{x}} - \overline{C}^\dagger(\mathbf{k})e^{-i\mathbf{k}\cdot\mathbf{x}}\Big]\,, \tag{12}$$

$$H_0 = -g^{\mu\nu}\int d^3k\,\omega(|\mathbf{k}|)a_\mu^\dagger(\mathbf{k})a_\nu(\mathbf{k}) + \int d^3k|\mathbf{k}|\Big\{\overline{C}^\dagger(\mathbf{k})C(\mathbf{k}) + C^\dagger(\mathbf{k})\overline{C}(\mathbf{k})\Big\}\,, \tag{13}$$

where

$$\omega(|\mathbf{k}|) = \sqrt{M^2 + |\mathbf{k}|^2}\,, \tag{14}$$

$$\Big[a_\mu(\mathbf{k}), a_\nu^\dagger(\mathbf{k}')\Big] = -g_{\mu\nu}\delta^3(\mathbf{k} - \mathbf{k}')\,, \tag{15}$$

$$\Big[a_\mu(\mathbf{k}), a_\nu(\mathbf{k}')\Big] = \Big[a_\mu^\dagger(\mathbf{k}), a_\nu^\dagger(\mathbf{k}')\Big] = 0\,, \tag{16}$$

$$\Big[\overline{C}(\mathbf{k}), C^\dagger(\mathbf{k}')\Big]_+ = \Big[C(\mathbf{k}), \overline{C}^\dagger(\mathbf{k}')\Big]_+ = \delta^3(\mathbf{k} - \mathbf{k}')\,, \tag{17}$$

$$\Big[C(\mathbf{k}), C_\nu^\dagger(\mathbf{k}')\Big]_+ = \Big[\overline{C}(\mathbf{k}), \overline{C}^\dagger(\mathbf{k}')\Big]_+ = \Big[C(\mathbf{k}), \overline{C}^\dagger(\mathbf{k}')\Big]_+ = 0\,. \tag{18}$$

The restriction of the Lorentz condition on a physical state $|\Psi_{ph}\rangle$ can be expressed as

$$k^\mu a_\mu(\mathbf{k})|\Psi_{ph}\rangle = 0 \qquad (k^0 = \omega(|\mathbf{k}|))\,. \tag{19}$$

For a single particle state

$$\varepsilon^\mu(\mathbf{k}, \lambda) a_\mu^\dagger(\mathbf{k})|0\rangle \tag{20}$$

where $\varepsilon^\mu(\mathbf{k}, \lambda)$ is the polarization vector, Eq. (19) gives

$$k_\mu \varepsilon^\mu(\mathbf{k}, \lambda) = 0\,. \tag{21}$$

According to this condition the longitudinal polarization ($\lambda = 3$) is allowed to be present together with two transversal polarizations ($\lambda = 1, 2$) for each $\mathbf{k}$.

Since the mass term in $\mathcal{L}_{eff}$ is replaced with $\widetilde{\mathcal{L}}_{AM}$ which is gauge invariant, the effective action $\int d^4x \mathcal{L}_{eff}(x)$ is invariant with respect to the usual BRST transformation

$$\begin{aligned}
&A_\mu(x) \to A_\mu(x) + \delta\zeta\partial_\mu C(x)\,,\\
&\overline{C}(x) \to \overline{C}(x)\delta\zeta\frac{1}{\xi}\partial^\mu A_\mu(x)\,,\\
&C(x) \to C(x)\,,
\end{aligned}$$

where $\delta\zeta$ is an infinitesimal fermionic real parameter independent of x. It is now particularly emphasized that if the three polarization states allowed by the condition (21) are really present, each of them should be invariant under the transformation

$$A_\mu(\mathbf{x}) \to A_\mu(\mathbf{x}) + \delta\zeta\partial_\mu C\,,$$

where $A_\mu(\mathbf{x})$ and $\partial_\mu C$ are operators in the Schrodinger picture. Correspondingly, the operators $a_\mu^\dagger(\mathbf{k})$ and $a_\mu(\mathbf{k})$ transform as

$$\delta a_\mu^\dagger(\mathbf{k}) = i\delta\zeta\widetilde{k}_\mu\sqrt{\omega/|\mathbf{k}|}C^\dagger(\mathbf{k})\,, \tag{22}$$

$$\delta a_\mu(\mathbf{k}) = i\delta\zeta\widetilde{k}_\mu\sqrt{\omega/|\mathbf{k}|}C(\mathbf{k})\,, \tag{23}$$

where the components of $\widetilde{k}_\mu$ are

$$\widetilde{k}_0 = |\mathbf{k}|\,, \qquad \widetilde{k}_l = k_l\,. \tag{24}$$

Therefore the BRST invariance of $\varepsilon^\mu(\mathbf{k},\lambda)a_\mu^\dagger(\mathbf{k})|0\rangle$ implys

$$\widetilde{k}_\mu \varepsilon^\mu(\mathbf{k},\lambda)C^\dagger(\mathbf{k})|0\rangle = 0\,. \tag{25}$$

Since $C^\dagger(\mathbf{k})$ are creation operators, one gets

$$\widetilde{k}_\mu \varepsilon^\mu(\mathbf{k},\lambda) = 0\,. \tag{26}$$

Actually, only the transversal polarizations satisfy this additional condition, thus the longitudinal polarization should be excluded.

In conclusion, a massive gauge boson, just like a massless one, has only two transversal polarizations.

It is clear that for a theoretical model which is formed from a gauge theory with various modifications, our arguments in this paper is still valid provided the gauge bosons in the initial and final states are described by the Lagrangian givin in Eq. (1). Consequently, we have much more room than before in constructing theoretical models.

In addition, we mention the unitarity-violating problem caused by the longitudinal polarisation. If the longitudinal polarisation were present, this problem could be found to exist based on the following relation

$$\sum_{\lambda=1}^{3} \varepsilon^\mu(\mathbf{k},\lambda)\varepsilon^{*\nu}(\mathbf{k},\lambda) = -g^{\mu\nu} + \frac{k^\mu k^\nu}{M^2}.$$

and on the fact that the longitudinal polarisation vcctor $\varepsilon^\mu(\mathbf{k},3)$ at high energy tends to k^μ/M. Such trouble, which in the pass were thought to exist, is one of big obstacles in developing massive gauge field theories. In our recent works, a massive SU(n) gague field theory was proved to be renormalisable [5] and a method was developed to construct the scattering matrix based on the renormalized theory [9]. Consequently, the exclusion enables a massive SU(n) gauge field theory to be logically consistent. This also indicates that it is naturally reasonable to exclude the longitudinal polarisation of a massive gauge boson.

Finally, for the known massive gauge bosons, we believe that they will be confirmed experimentally to have no longitudinal polarization.

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